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COMPETITION, REPRESENTATION AND REDISTRICTING

THE CASE AGAINST COMPETITIVE CONGRESSIONAL DISTRICTS

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ABSTRACT

A redistricting plan that maximizes the frequency of competitive elections does not maximize the representativeness of political outcomes. A plan more closely resembling a 'bipartisan gerrymander' (a non-competitive plan) out-performs a set of competitive districts. Drawing on the literature from democratic theory and the trade-offs between redistricting goals, I expand on previous models by taking legislators' and constituents' ideologies into consideration in order to compare the representativeness of a fair competitive and a fair non-competitive redistricting plan. I show that non-competitive districts lead to smaller ideological differences between the positions of district median voters and their representatives, voters being ideologically closer to their legislators in absolute terms, and a distribution of ideology in the legislature that is closer to the distribution of ideology in the electorate. Also, the competitive plan cannot simultaneously make legislators responsive to their district median voters while ensuring that there is a sufficient amount of variation in ideology among the state delegation. The non-competitive plan can meet all requirements. Consequently, I argue that competition is not as fundamental to representative government as we might think since competition does a surprisingly poor job of producing representative outcomes.

KEY WORDS • competition • congressional elections • redistricting • gerrymandering

Despite the relative lack of public attention to the subject, redistricting is one of the most contentious processes in politics. Partisan battles over redistricting are to be expected, but there is surprisingly little consensus about how district lines *should* be drawn from the perspective of democratic theory. Most texts on the subject suggest a number of competing goals to be addressed, such as partisan fairness, keeping communities of interest together, etc. Some of the literature suggests that there are explicit trade-offs between the various goals of redistricting (see, for example, Butler and Cain, 1992; Niemi, 1982; Niemi and Deegan, 1978). In general, though, scholars and

reformers agree that the creation of competition is an important goal to be pursued, or, that one side should not be structurally guaranteed a victory over the other. The purpose of this paper is to argue that competition is not necessarily a goal that should be pursued in the redistricting process. Competitive redistricting plans produce inherently unrepresentative outcomes, and what is the point of competition if not to achieve representative outcomes?

There is a long tradition in American political thought arguing that competition is the best way to achieve desirable political outcomes. In *The Federalist Papers*, Madison argues that government institutions should be designed to pit competing factions against each other so that no one faction can become dominant. Similar thoughts have echoed throughout our discourse, such as the APSA committee's arguments in *Toward a More Responsible Two Party System* that perpetual competition between the two parties was essential to the political health of the country. Most theorists seem to believe that if the results of an election are a foregone conclusion, that there is something wrong with the system. Competition, on the other hand, keeps democracy healthy, or so the line goes. Schumpeter (1942), for example, argues that by promoting competition in a market-like system, power is kept in check. Similarly, the 'marginality' hypothesis (which is questionable, as will be discussed later) suggests that candidates in electoral danger will be more likely to appeal to the median voter, thus fulfilling their representative functions (see Fiorina, 1973).

Nowhere are these types of arguments more prominent than in the debates over various redistricting plans. Redistricting plans such as the one implemented in California after the 2000 census protect incumbents by drawing districts that are relatively homogeneous with respect to the partisanship of their voters. Incumbents wind up representing districts in which 70 percent of the voters are of the same party as the incumbent, thus ensuring that no general election challenger will have a significant chance of winning. As a result, congressional elections are landslides. Even a cursory look at newspaper editorials will demonstrate that most people consider this state of affairs to be a bad one. In contrast, I argue that structuring our elections in such a way as to maximize competition does not uniformly serve democratic interests. While scholars have addressed the fact that the various goals of redistricting may come into conflict with each other, this paper will take a broader perspective and focus not on the trade-offs between various democratic goals, but on the inherent disparity between what competitive elections tend to achieve and what they are intended to achieve. While competition is theoretically intended to promote representativeness, a system of non-competitive elections may paradoxically lead to more accurate representation of ideological viewpoints.

Democratic Theory and Redistricting Goals: A Question of Representativeness

From the perspective of democratic theory, how important is it to draw district lines in a way that promotes competition? The literature on redistricting suggests a number of goals to be pursued, which often come into conflict with each other, yet it has generally been assumed that a redistricting plan that reduces the number of districts that are competitive is somehow 'undemocratic'. Why? What is it about competitive districts that makes them indispensable to democracy? In order to understand this issue, it is important for us to take a step back and talk about what democracy is. There is no single theory of democracy, and writings in democratic theory have suggested a wide array of goals, not all of which are directly relevant to redistricting. However, Shapiro (2003) distinguishes between two different types of functions necessary for democracy: aggregative and deliberative. While the latter function refers to institutional arrangements, Shapiro associates the former with Schumpeter's argument about the role of competition. Competition is necessary for democracy because it aggregates preferences. However, does competition really aggregate preferences in a representative way? That is the fundamental question of this paper.

So, what does it mean for preferences to be aggregated in a representative way? We must now formally define 'representativeness'. Achen (1978) provides a useful set of criteria to evaluate representativeness, but this paper will broaden those criteria slightly. Below I derive a set of four criteria with which to evaluate the 'representativeness' of a preference aggregation. These criteria will come from the combination of two sets of considerations.

First, Dahl (1956) suggests two types of theories for structuring a democratic system: a maximizing theory and a descriptive theory. Under a maximizing theory, one would structure the system in such a way as to maximally achieve a desired outcome. Under a descriptive theory,[1] one would structure the system in such a way as to ensure that the government has certain desired characteristics. The second set of considerations is the level at which the theory is applied. In the context of redistricting, we can apply maximizing theories and descriptive theories to the individual district level, and to the level of the legislature as a whole. Consequently, I consider four goals

1. Note that the use of the term 'descriptive' in this paper is slightly different from the use in most of the literature on redistricting. Most discussions of redistricting use the term when referring to the 'descriptive representation' achieved with the creation of majority–minority districts that allow minorities to elect legislators of their racial or ethnic group. The term will turn out to have a similar meaning in this paper, though, if we consider the issue of descriptive representation of ideological minorities.

based on the maximizing/descriptive distinction, and the district/legislature levels of analysis.

Goal 1: Minimizing the degree to which legislators deviate from the positions of the median voters of their districts

The first goal this paper will address is a maximizing goal applied to the district level. At the district level, the Median Voter Theorem (MVT) should force legislators to place themselves ideologically at or close to the positions of the median voters of their districts. Such an outcome maximizes the degree to which the legislator represents his district (if we assume equal intensity on all sides, but expanding the theory beyond that is an unnecessary complication). However, we know empirically that this does not uniformly happen. From a democratic theory perspective, such deviations from the median voter are a failure to achieve a key maximizing goal. Consequently, this paper will address whether drawing competitive district lines leads legislators to deviate more from their district median voters, or less from their district median voters.

Intuitively, it may seem obvious that having competitive districts would create incentives for legislators to be more responsive to the desires of their district median voters, along the lines of the marginality hypothesis mentioned earlier. Similarly, Bianco's (1994) work suggests that when legislators have the trust of their constituents (and thus are safe), they might feel less constrained to carry out the wishes of their constituents. However, the issue is not quite so clear cut. Recent work has demonstrated that when one candidate has a valence advantage (e.g. experience) over the other, despite having an electoral advantage, that candidate might position himself closer to the district median, not farther away (see, for example, Groseclose, 2001). Thus, there is precedent for the idea that making elections less competitive might bring legislators closer to the positions of their district medians.

Goal 2: Ensuring similarity between constituents and their representatives

The second goal this paper will address is a descriptive goal applied to the district level. While Goal 1 is about making sure legislators represent their districts optimally, Goal 2 will address how well such legislators are doing objectively when they behave optimally. Even when legislators are at the position of the median voter, there will be voters who are fairly far from their representatives, and the farther away voters are from their representatives in absolute terms, the less well the legislator can be said to represent them. Goal 2 is a descriptive goal that ensures that the optimal outcome of the election has descriptive meaning. An analogy might be made here to the distinction between a delegate and a trustee. Replicating the preferences

of the median voter is fulfilling the delegate functions of a representative, but a legislator's ability to serve as a trustee is dependent on how many interests he is entrusted with representing. If we think of a trustee not as someone elected to act in the best interests of the state (or country), but instead as someone elected to act in the best interests of his district, then the more broad the array of interests he is entrusted with representing, the more difficult his decisions will be. So, this paper will compare the performance of competitive and non-competitive districts in minimizing the mean squared deviation of constituent ideologies from their representative's ideology.

Goal 3: Minimizing the distance between the median voter of the state delegation and the median voter of the state

The third goal is a maximizing goal applied to the level of the state delegation.[2] Since policy decisions are made by elected officials, this goal is about ensuring that the result of policy-making processes will be identical to what it would be if voters decided directly. This paper will compare the performance of competitive and non-competitive districts in minimizing the distance between the state's median voter and the median voter of the state delegation.

Goal 4: Ensuring that the distribution of preferences in the state delegation is representative of the distribution of preferences in the state electorate, or, proportional representation of ideological viewpoints

Goal 4 is a descriptive goal applied at the level of the state delegation. Goal 3 was about ensuring that the policy outcomes in the legislature were representative. However, suppose a legislature does not merely exist to replicate the wishes of the median voter as often as possible. Suppose, for the moment, that there is value to having a broader array of interests represented in a legislature so that the legislature is descriptively representative of the public as a whole. Like Goal 2, this is a descriptive goal aimed at ensuring not just that the median voter of the state delegation represents the median voter of the district, but that even those who are relatively far from the position of the median voter have some representation. We can think of this as a sort of 'proportional representation' requirement for ideological viewpoints. In fact, there is considerable evidence from the literature on

2. When discussing aggregate qualities of a redistricting plan, this paper will refer to the state delegation, which is the set of all legislators elected from the state being redistricted. It could just as easily refer to an entire state legislature, or any other political unit.

redistricting that competitive district lines do not always produce proportionately representative outcomes. Butler and Cain (1992), for example, demonstrate that, empirically, competitive redistricting plans can produce larger seat-vote gaps than bipartisan gerrymanders. Formally, Niemi and Deegan (1978) and Niemi (1982) argue that there is a direct mathematical trade-off between competitiveness and partisan balance. Given that, it should not be all that surprising if competitive redistricting plans do a poor job of producing a state delegation that is ideologically representative of the population.

While this goal has an appeal to democratic values on its face, some of its practical consequences may not be entirely popular. Having a wide range of interests represented also means that there are limits to the legislature's ability to come to agreement. Increasing the ideological variance within the legislature, after all, goes by another name: 'polarization'. The more highly polarized the legislature is, the more contentious the process of legislating will be, and that is something the people generally do not like (Hibbing and Theiss-Morse, 1995). However, people's dislike of contentious legislative processes is not something that is crucial to address here. This paper is about broad goals derived from democratic theory, and tolerance for the 'sausage-making' aspects of legislative politics does not rise to that level of concern. The broad theory of concern here is the issue of representativeness. This paper will compare the performance of competitive and non-competitive districts in creating a state delegation whose ideological distribution looks like the ideological distribution of the state, or, the performance of different plans in achieving ideological proportional representation.

It should be noted that the Supreme Court has ruled that the goal of achieving 'proportional representation' is not constitutionally required (Davis v. Bandemer, White v. Regester), but that does not mean it is not an appropriate goal to achieve.

The four goals derived above are briefly restated and categorized in Table 1. I do not claim that this is an exhaustive, nor even a particularly thorough list of all democratic goals.[3] However, they do provide a fairly complete picture of what it means to aggregate preferences in a representative way with respect to ideological viewpoints. Thus, they provide an appropriate way to compare how representative outcomes are under competitive and non-competitive redistricting plans. With the goals derived above, we now have a set of criteria that can be used to analyze the representativeness of any given redistricting plan. Next, the paper will set up a model, and discuss two hypothetical redistricting plans: a competitive redistricting plan, and a non-competitive redistricting plan.

3. Interestingly, Achen (1978) claimed that a slightly narrower set of goals was exhaustive as far as mathematical specifications of issues in representation.

Table 1. Type of Goal

Level	Maximizing	Descriptive
District	Goal 1: Minimizing the distance between legislators and the median voters of their districts	Goal 2: Minimizing the gaps between voters and their reps
State delegation	Goal 3: Minimizing distance between med. voter of the delegation and med. voter of state	Goal 4: PR: Insuring that the ideo. distrib. in the state delegation is similar to the distrib. among voters

The Model

Assumptions

(I) Unidimensional ideology. Each voter in a state to be redistricted has an ideology score along a single dimension. This score corresponds to an ideal point with single-peaked, symmetric preferences.

(II) Population. The state has $n * m$ people where n is the number of districts that need to be drawn and m is the number of people to be in each district.

(III) Normally distributed ideal points. Each voter's ideology score is a random draw from a standard normal distribution. Since n^*m is an arbitrarily large number, the distribution of ideal points is essentially continuous.[4] The assumption that the voters in a state are, on average, perfectly moderate (i.e. that the distribution has a mean of 0, as is the case with a standard normal distribution) does not affect the substance of the model, except with respect to one issue. It allows us to draw competitive districts by making them microcosms of the state. This is a minor point that will be addressed in footnotes.

(IV) Range restriction. A district cannot either nominate or elect a legislator whose ideology is outside the ideological range of a district (which will be finite when districts are truncated sections of a normal distribution).

(V) Flexible district shapes. District lines can group constituents based on ideology without external constraints such as compactness requirements.[5]

(VI) FPTP/Two-party system. Elections are first-past-the-post. Consequently, there are two parties: a left party and a right party.

4. The fact that ideology is unbounded in this model is substantively unimportant since there are so few voters in the tails that they do not present an analytic problem.

5. As in Niemi and Deegan (1978), this assumption is necessary since mathematical definitions of such requirements are unnecessarily difficult to incorporate into an abstract formal model without specified geography. The practical consequences of this assumption will be addressed in the course of the paper.

(VII) Party cohesion. A left party candidate cannot run with a positive ideology score, and a right party candidate cannot run with a negative ideology score.[6]

(VIII) Sincere voting. All voting is sincere (i.e. even in a primary, voters will choose candidates closest to their ideal points, regardless of whether or not that candidate is likely to win the general election).[7]

Election Processes

The election process this paper will use is a sequential election model with a primary and a general election. The process is as follows. In every district, each party has a primary electorate which is a sub-sample of the district population. The left party's primary electorate has a median that is to the left of the district median, and the right party's primary electorate has a median that is to the right of the district median. These primary medians are equidistant from the district median when the distribution of voters is symmetric and centered at 0. Suppose that when the district has a standard normal distribution, this distance is a constant, D, where $D > 0$.[8] Two candidates select fixed issue positions on which to run in each primary, then the whole district decides between the two primary winners.

This model is not based on any specific primary rules (such as an open or closed primary), but simply assumes that primary electorates are ideologically biased.[9] The concept of the open primary[10] is that it makes the primary electorates more similar to the overall electorate, thus reducing the bias. It is empirically uncertain how much open primaries reduce the bias, but it is highly unlikely that they eliminate the bias. Theoretically, though, they are intended to force nominees to be closer to their district medians.

6. At some points in history, this assumption might seem problematic. However, in the current era of party polarization, there is very little ideological overlap among Members of Congress, as any analysis of NOMINATE scores will demonstrate. Furthermore, as Cox and McCubbins (1993) argue, parties attempt to promote a 'brand name' which has very little meaning without party cohesion, giving them additional incentives to promote party cohesion. In the long run, a state's median may shift, and parties may shift with them. However, this can be incorporated into the model by simply having the 'zero' point shift along with the population. After all, having a distribution centered at zero is simply a mathematical convenience anyway.

7. Strategic voting in congressional primaries is probably not a realistic assumption given the level of political sophistication it requires.

8. In fact, given the two assumptions about primary medians, it is necessary that $D > 0$ when districts are symmetric and centered at 0 because primary medians must be more extreme than district medians.

9. Note, though, that the model also assumes that the general electorate is unbiased. See Wolfinger and Rosenstone (1980).

10. An open primary is one in which voters choose a party, and vote in the primaries for that party regardless of their registration.

Interestingly, if this happens, the competitive plan will fulfill Goals 1 and 3 very effectively, but it will fail to fulfill Goal 4, as we shall see.

Note that in this process, candidate ideology scores are fixed once chosen, which may not be empirically true. However, if candidate ideologies are not fixed, the concept of representation itself would seem inappropriate. Representation requires that there be some sort of commitment to the positions taken during a campaign, which is a key to the brand name interpretation that Cox and McCubbins (1993) discuss. Furthermore, as Downs (1957) argues, voters may punish those who do not hold to those commitments anyway. Thus, there is sufficient justification for treating candidate positions as fixed. However, again, even if we believe that the assumption is unrealistic, this paper will consider both the possibility that candidates can take the positions of their district medians, and the possibility that they are constrained by their primary constituencies, so the gamut of possibilities will be addressed.

Competitive Elections

We must now have a working definition of a 'competitive' election. Specifically, this paper is concerned with competition at the level of the general election. Since this is a deterministic spatial voting model and all voters have single-peaked, symmetric preferences, in any given election, whichever candidate is closer to the position of the median voter will win with probability 1. If both candidates are equidistant from the median, each candidate will win with probability .5. Thus, I will define an election as 'competitive' when either candidate has a .5 probability of winning, and I will define an election as 'non-competitive' when one candidate wins with probability 1. Similarly, I will refer to a redistricting plan as a competitive plan if it promotes competitive *general* elections, and I will refer to a plan as non-competitive if it promotes non-competitive *general* elections.

Redistricting Plans

Given the assumptions and election processes described above, I will now specify two sample redistricting plans, one of which will produce competitive elections, and the other will not. These examples are not intended to cover the range of all possible redistricting plans. They are intended to serve as examples of plans that either have or lack competitive elections, but are both 'fair' since they are unbiased. The objective for the paper is to show that the fair but non-competitive plan produces more representative outcomes than the fair, competitive plan. Since I am only concerned with comparing fair plans, this paper will not address the use of efficient partisan gerrymanders (i.e. one in which the planner maximizes the seat gain for one party). This strategy is certainly anti-competitive since it either spreads

the disadvantaged party's voters so thinly that they cannot win any districts or packs them so tightly into a few districts that they cannot win any others (or some combination of the two). However, such plans are difficult to draw and risky to attempt. Consider, for example, the failed attempt by New York Republicans to take over the legislature that was thwarted by the post-Watergate backlash (Scarrow, 1981). More importantly, such plans are inherently biased (which is, after all, the point), and the purpose of this paper is to show that competitive redistricting plans are not the most representative, which only requires me to demonstrate that there is *one* type of non-competitive plan which produces more representative outcomes than the competitive plan. In that respect, the partisan gerrymander is a straw man, and is therefore irrelevant.

A Competitive Redistricting Plan Consider the following as an exemplar of a fair redistricting plan that maximizes the number of competitive elections: each district will be a perfect microcosm of the state as a whole. Each district, then, will be m random draws from the state, and like the state, each district will have a distribution of ideology scores that is virtually a continuous standard normal distribution. Each district will be ideologically heterogeneous with an expected variance of 1, an expected mean and median of 0. Now, consider some arbitrary point, D^*, where $0 \leq D^* \leq D$.[11] Any combination of strategies in which the two left party candidates position themselves at $-D^*$ and the two right party candidates position themselves at D^* will be a Nash equilibrium. All candidates are equidistant from their primary medians, and thus have a .5 probability of winning the primary, and the winners are equidistant from the district median, and thus have a .5 probability of winning the general election. Thus, each candidate has a .25 probability of winning. If any candidate deviates from this equilibrium by moving towards his district primary median, that candidate will deterministically win the primary, but deterministically lose the general, so there is no incentive to move towards the district primary median. If any candidate deviates from this equilibrium by moving towards the district median, that candidate will deterministically lose the primary, so there is no incentive to move towards the district median. Thus, as long as $0 \leq D^* \leq D$, this is a Nash equilibrium, and all elections are competitive. Such a redistricting plan maximizes competition in that it maximizes the number of districts that are competitive.[12]

11. Recall that D is the distance between a primary electorate's median voter and 0 when the distribution of voters is a standard normal distribution.

12. If the state is not ideologically centered at 0, we can draw q districts where $q < n$ which have standard normal distributions centered at 0 while placing the rest of the voters into a set of non-competitive districts. That would maximize the number of competitive elections. Mathematically, this generates substantively identical results to what will be discussed later.

The important substantive characteristic here is that this plan creates competitive districts by ensuring that each district contains voters with a fairly wide range of ideological viewpoints.

However, this means that there are an infinite number of Nash equilibria. So, what equilibrium should we expect? Let d be the distribution of ideology scores of the winners under the competitive plan, and let d_i be the legislator's ideology score from district i. Based on the requirements of a Nash equilibrium here, we know that d_i can be anywhere in the following range: $[-D, D]$. It is possible that $d_i = 0$ for any given district. It is even possible that $d_i = 0$ for all i, in which case everyone simply takes the position of the district median voter.[13] However, there are a number of compelling reasons to believe that $E(|d|) > 0$, or, that legislators should be expected to deviate from their district median voters. In fact, there are compelling reasons to believe that $E(|d|)$ is substantively large. First of all, we know empirically that legislators do not simply place themselves at the positions of their district medians. Formally, even Downs (1957) does not argue that parties will place themselves at exactly the position of the median voter due to concern over extreme voters abstaining. The most compelling reason to believe that legislators deviate from their district medians, though, is that Mayhew's (1974) assumption that parties and candidates only care about winning is wrong. As Fenno (1973) argues, candidates care primarily about three things: winning elections, institutional power, and policy. Furthermore, as Calvert (1985) and Wittman (1977, 1983) demonstrate, if candidates are uncertain about voters' preferences and have policy preferences of their own which are more extreme, they will take positions that are not those of their district medians. This model permits legislators to deviate from their district medians by as much as D under the competitive plan because the primary electorates are politically different from the general electorates, along the lines of Aldrich's (1995) argument. Furthermore, we know Members of Congress have polarized, bimodally distributed ideological preferences based on such measures as NOMINATE scores. Thus, suppose candidates also have bimodally distributed ideology scores. In that case, suppose that a legislator's ideology score from an arbitrary competitive district is a draw from the following distribution:

$$d\begin{cases} d_i = d' + \varepsilon & \text{with prob. of } .5 \\ d_i = -d' + \varepsilon & \text{with prob. of } .5 \end{cases} \quad \text{where } d' > 0,\ |d_i| \leq D \text{ and } E(\varepsilon) = 0$$

13. In this case, the competitive plan would fulfill Goals 1 and 3 perfectly, but as we shall see, this would cause problems for the competitive plan with respect to Goal 4.

This distribution is based on the assumption that congressional candidates have ideology scores that have a similar distribution to Members of Congress. Groseclose's (2001) results and others suggest that, if anything, losing candidates are even *more* extreme than winning candidates, so this distribution of d is a theoretically and empirically justifiable distribution. Where relevant, though, I will discuss the implications of having a different distribution of d (where d must always be bounded by the range of $[-D, D]$).

A Non-competitive Redistricting Plan Under a redistricting plan that minimizes the number of competitive elections, each district's ideological distribution will be all observations from a truncated section of the standard normal distribution based on the following requirements (with each district drawn from a different section):[14] If F is the cumulative distribution function (CDF) for a standard normal distribution, a line of truncation is placed at $x_1 = F^{-1}(1/n)$, $x_2 = F^{-1}(2/n)$, and so forth. For example, if there are two districts, the truncation line is placed at $x = 0$. The first district would be the set of all voters with negative ideology scores, and have a distribution that looks like a truncated standard normal distribution bounded at the right by 0. The second district would be all voters with positive ideology scores, and have a distribution that looks like a truncated standard normal distribution bounded at the left by 0. If there are four districts, the truncation lines would be placed at $-.67$, 0 and .67, and each district would be the people in one of the non-overlapping sections.

Because of the combination of the range restriction (Assumption IV) and the party cohesion restriction (Assumption VII), no non-central district[15] can nominate candidates of the out-party (the party with ideology scores the opposite sign of the voters in the district).[16] Consequently, each general election in a non-central district will have only one candidate (positioned at the in-party's primary median), and will thus be non-competitive. Thus, there will be either zero, one, or two competitive districts, as discussed below.

If the number of districts is odd, there will be one competitive district (the central district). If the number of districts is even, there will be either zero or two competitive districts (the central districts). In the central districts, the out-party's candidates will place themselves at 0. If 0 is farther from the district median than the in-party's primary median, both in-party candidates

14. These requirements will ensure that each district has a population of m.

15. A non-central district is one in which 0 is not included in the range of ideology scores or as an endpoint.

16. This should not be troubling since there are a large number of urban districts that are so heavily Democratic that Republicans do not even nominate candidates to run in those races. In other districts with strong incumbents, frequently the other party will only put up a sacrificial lamb as a candidate that they know will lose. Empirically, this result makes sense.

will place themselves at their primary medians, the in-party's nominee will then be closer to the district median than the out-party candidate, and the general election will be non-competitive. If 0 is equidistant from or closer to the district median than the in-party's primary median, the in-party's candidates could place themselves at the same distance from the district median as the out-party's candidates, and the general election will be competitive, with each candidate having a .25 probability of winning. This is a Nash equilibrium because neither out-party candidate can deviate at all, and neither in-party candidate has an incentive to deviate. If either in-party candidate moves closer to the district median, he loses the primary deterministically. If either in-party candidate moves closer to the primary median, he would lose the general election deterministically. Thus, the two central districts might be competitive in the general election.[17] Competition is thus minimized. Substantively, the important point is that this plan minimizes the number of competitive districts by minimizing ideological heterogeneity within each district (as opposed to the competitive plan).

Such a redistricting plan strains credibility. No state has sufficiently precise data with respect to ideology to be able to draw such districts (not to mention the flagrant violations of compactness that would be entailed). However, the truncated shapes of the distributions are not what drive the results of this model. As the paper will discuss throughout the analysis, the key features that drive the results are that the districts have medians that are shifted away from the median of the state and that the districts have smaller variances in ideology scores than the state. Truncating the distributions, however, greatly simplifies the math so that the focus can be on the substance rather than the calculus. Furthermore, mathematically specifying districts with shifted medians and smaller variances is tricky because there is no unique method for doing so, and no obvious criterion for selecting a method. For the purposes of this paper, truncated districts will suffice.

Furthermore, substantively, this plan resembles very closely an example of a bipartisan gerrymander. Under such a plan, district lines are drawn in order to minimize the number of districts that are marginal. The intended effect and practical consequence of this plan is to protect incumbents by ensuring that their districts do not have enough voters of the other party to mount a credible campaign in the general election. Realistically, those drawing district lines certainly seek a balance between taking partisan advantage of the process and protecting incumbents, but the goal of this paper is not to compare

17. Note also that it is possible for the left party candidate to lose the left-center district, and for the right party candidate to lose the right-center district. This allows for the possibility of Republicans to occasionally win in left-leaning districts, and Democrats to win in right-leaning districts. Thus, an important empirical phenomenon is captured by this model.

actual plans. It is to show that maximizing competition is not an optimal approach based on democratic theory because there is another plan, which is non-competitive, that would lead to more representative outcomes.

Competition and Heterogeneity

By construction, this model is one in which competitive elections result from ideological heterogeneity within districts. However, there has been some debate over whether heterogeneity does empirically lead to competitive elections. Bond (1983), for example, argues that constituent heterogeneity does not translate to increased competitiveness. However, Koetzle (1998) demonstrates that while demographic heterogeneity does not inherently produce competition, political heterogeneity does. This model is based on political (ideological) heterogeneity rather than demographic, and is thus supported by empirical findings.

Performance of the Two Redistricting Plans with Respect to Democratic Goals

Goal 1: Minimizing the degree to which legislators deviate from the positions of the median voters of their districts

The issue that one might consider most problematic with a system that lacks competition in legislative elections is that an incumbent may deviate from the median voter of his district and still continue to win re-election. Reformers and normative theorists generally consider such deviations to be undesirable. The presence of competition might be seen as a constraint on legislators' behavior to keep them from deviating from the median voter along the lines of the marginality hypothesis. However, in a sense, this problem results from ideological heterogeneity within congressional districts.

Let us first consider the competitive redistricting plan. Recall that under the competitive redistricting plan, winning candidates can have ideology scores anywhere in the range $[-D, D]$. Consequently, it would be a Nash equilibrium for a legislator under the competitive plan to have an ideology score of 0, which is his district median. In fact, it is even theoretically possible that every legislator under the competitive plan will have an ideology score at his district median. If that were the case, then the competitive plan would perfectly satisfy Goal 1 (and, as we shall see, have problems satisfying Goal 4). However, for the reasons discussed earlier, this is not likely. What is more likely is that $E(|d|) > 0$, or that on average, legislators under the competitive plan will deviate from their district medians by some substantively significant

amount, regardless of the distribution of d. So, how do such deviations compare to deviations under the non-competitive plan? Do legislators from non-competitive districts deviate more from their district medians than $E(|d|)$, or do they deviate less from their district medians than $E(|d|)$?

Under the non-competitive plan, recall that candidates place themselves at the positions of their primary medians because there is no general election challenger (except in a few possible cases). Since primary medians are more extreme than district medians, this means that candidates under the non-competitive plan systematically deviate from their district median voters. Since candidates under the competitive plan could potentially place themselves at their district medians, it might appear at first glance that the competitive plan out-performs the non-competitive plan with respect to Goal 1. Again, though, it is more likely that a legislator from a competitive district deviates from his district median by some amount, $E(|d|)$ on average, which is bounded above by D. So, what is the upper bound on how much legislators from *non*-competitive districts can deviate from their district medians?

The tactic for this section of the paper will be to estimate the maximum possible distance a legislator from a non-competitive district can be from his district median voter in a non-tail district.[18] I will refer to this distance as *maxdist* in order to avoid excessive wordiness. Then, the paper will ask whether or not it is reasonable to believe that $maxdist > D$ or $maxdist < D$, and hence whether deviations from district medians should be larger under the competitive plan or under the non-competitive plan. If $maxdist < D$, then the cap on deviations is more restrictive under the non-competitive plan, and the non-competitive plan out-performs the competitive plan.

As an example, let us consider a state that is to be divided into 10 districts. The non-competitive plan creates 10 truncated normal distributions and groups the voters from each truncated section into the same district. In order to ensure that each of the 10 districts has approximately m people, districts are created with cut-points at the following values of ideology: -1.28, $-.84$, $-.52$, $-.25$, 0, .25, .52, .84, 1.28. Thus, each district has the shape of a truncated section of the standard normal distribution, and is bounded at the left and right with the values shown in Table 2.

Under the non-competitive plan, winners are at the position of their district primary medians, which are more extreme than their district medians. Thus, they deviate from the positions of their median voters. How much can they possibly deviate under the non-competitive plan? In the non-competitive plan, the districts are truncated, which places a cap on the

18. Distances are unbounded in tail districts.

Table 2. Ideological distributions in non-competitive districts for a state with 10 districts

District	Lower bound	Upper bound	Median	Max. distance from median
1	$-\infty$	−1.28	−1.64	undefined
2	−1.28	−.84	−1.04	.24
3	−.84	−.52	−.67	.17
4	−.52	−.25	−.39	.14
5	−.25	0	−.13	.13
6	0	.25	.13	.13
7	.25	.52	.39	.14
8	.52	.84	.67	.17
9	.84	1.28	1.04	.24
10	1.28	∞	1.64	undefined

maximum distance a voter can possibly be from the median voter, and hence places a cap on the maximum possible distance a representative can be from the median voter based on Assumption IV (except in the tail districts, which can potentially have voters at any distance from the median).[19] Referring back to Table 2, the largest possible distance from the median voter that a voter can be in a non-tail district is .24, which can occur in districts 2 and 9. Thus, when $n = 10$, *maxdist* is .24. That is a very small maximum distance. If F is the CDF of a standard normal distribution, $F(.24) - F(-.24) = .18$. In the context of a standard normal distribution, .24 is such a small deviation that draws from a standard normal distribution will be further than that from the mean 82 percent of the time, so it is certainly reasonable to believe that $D > .24$, and even that $E(|d|) > .24$. Thus, if there are 10 districts, legislators from non-competitive districts are likely to be closer to their district median voters than legislators from competitive districts.

We can also generalize to a plan with n districts. Clearly, *maxdist* will occur in the districts with the largest range, which will be the districts adjacent to the tails. Without loss of generality, let us examine the left-most non-tail district. This district will have a range with a size of $F^{-1}(2/n) - F^{-1}(1/n)$, and the district median will be at $F^{-1}((1/n) + (1/2n)) = F^{-1}(3/2n)$. *Maxdist* will be the distance between the median and the lower (left) bound of the district, or, $maxdist = F^{-1}(3/2n) - F^{-1}(1/n)$. With a simple thought experiment, it can then be shown that as n approaches infinity, *maxdist* approaches 0.

19. Recall that one of the assumptions was that a district could not elect anyone whose ideology is outside the bounded range of the district's ideology. Hence, the maximum distance between the median voter's ideology and an elected representative's ideology is the maximum distance between the median voter's ideology and the furthest endpoint of the district.

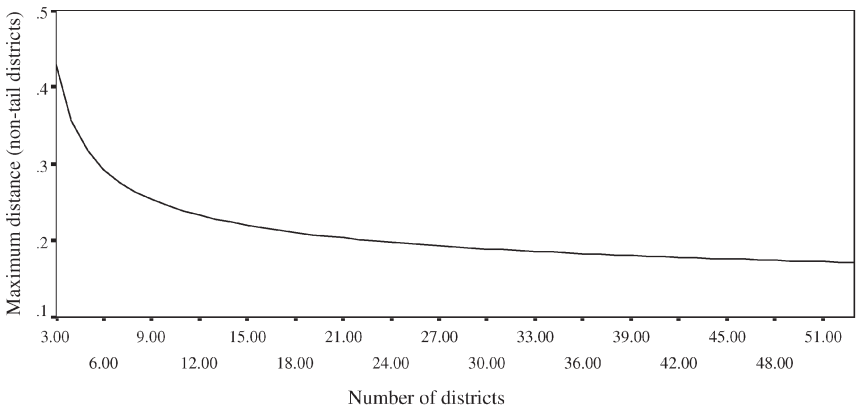

Figure 1. Maximum Possible Distance a Legislator Can Be from the District Median Voter under Non-competitive Elections

Consider the hypothetical case of an infinite number of districts. The area under the curve for each district will be $1/\infty$, so the distance between the upper and lower bounds of each district must be infinitesimal. As a result, the distance between the median and the lower bound (*maxdist*) will also be infinitesimal. Also, *maxdist* decreases monotonically as the number of districts increases, or, $d(maxdist)/dn < 0$. Proving this proposition algebraically is exceptionally tedious since $F^{-1}(x)$ does not have a closed form algebraic representation, but we can plot *maxdist* for any given given number of districts, which is done in Figure 1 up to 53 districts – the current number of House districts in the largest state (California).[20] The graph clearly shows that as the number of districts increases, *maxdist* decreases when districts are drawn to minimize competition. Along similar lines, we can also plot the probability that a random draw from the state will be further from the state median (0) than *maxdist*, which is done in Figure 2.

The probability that a random voter from the state will be further from the state median than *maxdist* under the non-competitive plan very quickly goes above .8, and approaches 1 since *maxdist* approaches 0, so the distance between legislators and their district medians very quickly becomes substantively insignificant as the number of districts increases. As the number of districts increases, the non-competitive plan performs increasingly well in minimizing the degree to which legislators can deviate from the positions

20. Such plots can also be generated for any number of districts, such as 400 (the number of districts in the New Hampshire House – the largest state legislature in the country), and the results are the same. Since the finding holds for any meaningful value of n, a formal proof is not substantively necessary.

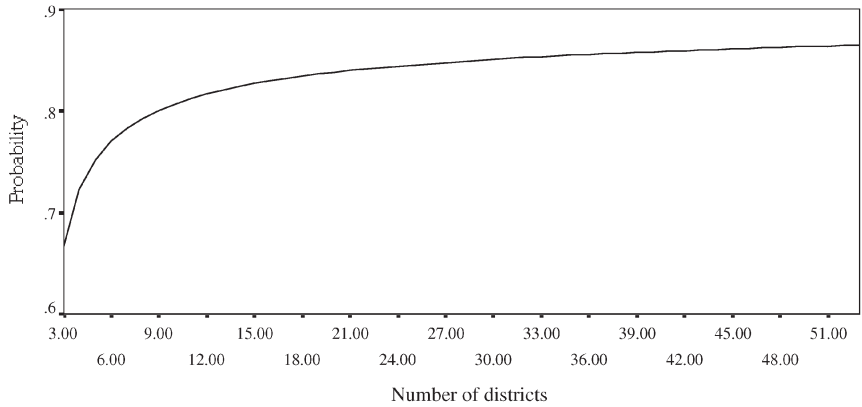

Figure 2. Probability that a Random Voter Is Further from the State Median than the Maximum Distance under the Non-competitive Plan

of their median voters. Furthermore, it clearly out-performs the competitive plan. After all, a random voter is highly likely to be further from the state median than *maxdist* under the non-competitive plan. Since *maxdist* very quickly becomes substantively insignificant as the number of districts increases, it is clearly reasonable to believe that $maxdist < E(|d|)$, and that $maxdist < D$ for essentially any number of districts and for any reasonable distribution of d.[21] Thus, while the competitive plan allows legislators to position themselves at their district medians, legislators are actually likely to deviate more from their district medians under the competitive plan than under the non-competitive plan.[22]

One might point out, though, that $E(d) = 0$ under the competitive plan, so it is unbiased. Interestingly, suppose d_i is the difference between a representative and his district median in district i under the non-competitive plan. $E(d_i) < 0$ if $i < n/2$, and $E(d_i) > 0$ if $I > n/2$. So, representatives are biased to the left in left districts, and they are biased to the right in right districts. Why should we prefer smaller deviations if it requires a bias? In fact, even under the non-competitive plan, $E(d) = 0$ when we look at all districts together because each district has a mirror which should have an equal bias in the opposite direction. So, the non-competitive plan out-performs the competitive plan with respect to Goal 1.

21. Again, a reasonable distribution of d is one in which $E(|d|) > 0$ based on what we know from previous research.

22. Again, though, keep this finding in mind because even if the competitive plan meets Goal 1, it will fail to meet Goal 4, but the non-competitive plan can meet both.

Note, though, that while the demonstration of this proposition relied on the truncated shape of the districts in the non-competitive redistricting plan, the principle does not. The effect comes from the fact that non-competitive districts have smaller variances in ideology scores than competitive districts. Suppose we take sample 1: n draws from $N(\mu, \sigma_1^2)$ distribution, and sample 2: n draws from a $N(\mu, \sigma_2^2)$ distribution. Suppose, also, that $\sigma_2^2 > \sigma_1^2$. If we take biased sub-samples from samples 1 and 2 (sub-samples 1^* and 2^* respectively) such that $p(x_{selected}) = g(x)$ where $dg(x)/dx > 0$,[23] the mean of sub-sample 1^* will be closer to μ than the mean of sub-sample 2^*. Observations far from μ in either sample 1 or sample 2 are equally likely to be selected, but since there are fewer observations in sample 1 that are far from μ that can possibly be selected since $\sigma_2^2 > \sigma_1^2$, there will be fewer outliers in sub-sample 1^* to pull the mean of sub-sample 1^* away from μ. More intuitively, if a district is more homogeneous, there are fewer outliers that can possibly pull the primary electorate far from the median of the district. Consequently, the median voter in a primary will be closer to the median voter in the general election. So, homogeneous districts will elect legislators closer to their median voters than heterogeneous districts.

Goal 2: Ensuring similarity between constituents and their representatives

The analysis in the previous section demonstrated that representatives from non-competitive homogeneous districts will generally be closer to their median voters than representatives from competitive heterogeneous districts. When a representative is ideologically identical to the median voter, that representative is behaving in accordance with the preferences of his district to the highest degree possible. However, it is easier to represent homogeneous districts than it is to represent heterogeneous districts. We might consider the mean squared deviation from the representative for a given district as a measure of how close voters in a district are to their representative. We can evaluate a redistricting plan based on the mean squared deviation of all voters from their own representatives (with lower mean squared deviations meaning better representation). The following point is tautological, and requires no extended proof. Districts with smaller variances will have voters that are ideologically closer to each other, and hence, closer, on average, to their representatives. Consequently, voters in districts that are more homogeneous will be, on average, closer to their representatives. Since the

23. The probability of a person with an ideology score of x being selected into a sub-sample (i.e. voting in the primary) is an increasing function of conservatism. In this case, g would represent the probability of voting in the Republican primary since it increases as ideology increases.

non-competitive redistricting plan creates districts with smaller variances in ideology scores than the competitive redistricting plan, the non-competitive plan out-performs the competitive redistricting plan with respect to ensuring that districts are descriptively representative.

Goal 3: Minimizing the distance between the median voter of the state delegation and the median voter of the state

The previous two goals focused on the relationship between constituent ideologies and legislator ideologies within each district. However, representative democracy requires not just that each district is appropriately represented, but that the delegation as a whole reflects the state at large. Thus, it is not just important that individual legislators accurately represent their constituents, but that the group of legislators accurately represent their state as a whole. So, it is important that the median voter in the state delegation reflect the median voter in the state. Otherwise, policy outcomes will not reflect the distribution of voters' preferences.

Consider the competitive plan. Again, it is possible for each candidate to position himself at his district median voter since this is a Nash equilibrium, and if this is the case, then by necessity, the median legislator in the state delegation will be at ideology 0, which is the state median. Hence, the state delegation would have the same median as the state electorate. Under those circumstances, the competitive plan would fulfill Goal 3 (but again, fail to fulfill Goal 4, as we shall see). However, recall that it is more likely to believe that each legislator's ideological position is d_i where d_i is a random draw from the following distribution:

$$d\begin{cases} d_i = d' + \varepsilon & \text{with prob. of .5} \\ d_i = -d' + \varepsilon & \text{with prob. of .5} \end{cases} \quad \text{where } d' > 0, |d_i| \leq D \text{ and } E(\varepsilon) = 0$$

If that is the case, then it may actually be impossible for any legislator to have an ideology score of 0. If $\max(|\varepsilon|) < d'$, then it is impossible for any legislator to have an ideology score of 0, and hence impossible for the median legislator to have an ideology score of 0. It is not a particularly bothersome assumption that $\max(|\varepsilon|) = d'$, but it is an additional assumption that is necessary in order for the competitive plan to even be able to fufill Goal 3. However, suppose $\max(|\varepsilon|) = d'$. Let p be the probability that the median legislator under the competitive plan will be within ω of 0 where ω is arbitrarily small. We can say that p is defined by the following function: $p = h(d', n, \sigma_\varepsilon^2)$. Furthermore, $dh/dd' < 0$, $dh/dn > 0$, and $dh/d\sigma_\varepsilon^2 > 0$. As d^1 goes up, the probability that the competitive plan will fulfill Goal 3 goes down, and as σ_ε^2 goes up, the probability that the competitive plan fulfills Goal 3 goes up. More importantly, as the number of districts goes up, the probability goes up. Thus, as

the number of districts increases, the competitive plan performs increasingly well with respect to Goal 3. However, this only happens probabilistically, and only with an additional assumption. Of course, if d had a different distribution, the functions would change, but the logical problem is the same. The competitive plan will still only meet Goal 3 probabilistically, and it will be dependent on the functional form of the distribution of d. Therefore, the competitive plan might meet Goal 3, but only weakly.

Under the non-competitive plan, if the number of districts is odd, the median legislator will come from the central district, which will elect a legislator whose ideology must converge to 0 as the number of districts increases (again, see the discussion of Goal 1). This requires no additional assumptions and happens deterministically, so if the number of districts is odd, the non-competitive plan clearly out-performs the competitive plan with respect to Goal 3.

Now, let us consider what happens when the number of districts is even. The two central districts will elect the two median legislators, and they will be equidistant from the median of the state electorate, so the average will be 0. Furthermore, it can be shown that the distance between these two median legislators approaches 0 as n increases. Due to the range restriction, that distance will be bounded above by the distance between the lower bound of the left-central district and the upper-bound of the right-central district. The lower bound of the left-central district will be $F^{-1}((1/2)-(1/n)) = F^{-1}((n-2)/2n)$, and the upper bound of the right-central district will be $F^{-1}((1/2)+(1/n)) = F^{-1}((n+2)/2n)$. Thus, the distance between the two median legislators is bounded above by $F^{-1}((n+2)/2n) - F^{-1}((n-2)/2n)$. Furthermore, the same thought experiment used in the discussion of Goal 1 can demonstrate that

$$\frac{d\left(F^{-1}\left(\frac{n+2}{2n}\right) - F^{-1}\left(\frac{n-2}{2n}\right)\right)}{dn} < 0$$

or that as the number of districts increases, the cap on the distance between the two median legislators gets smaller, and that

$$\lim_{n\to\infty}\left(F^{-1}\left(\frac{n+2}{2n}\right) - F^{-1}\left(\frac{n-2}{2n}\right)\right) = 0$$

or, as the number of districts approaches infinity, the cap on the distance between the two median legislators approaches 0. Thus, when the number of districts is even, the ideologies of the two median legislators deterministically converge to 0 with no additional assumptions. Again, the non-competitive plan out-performs the competitive plan with respect to Goal 3.

The non-competitive plan is more likely to produce a median legislator, or median legislators, who are closer to the position of the median of the state electorate than the competitive plan.

Goal 4: Ensuring that the distribution of preferences in the state delegation is representative of the distribution of preferences in the electorate

Suppose a legislature does not merely exist to replicate the wishes of the median voter. Suppose, for the moment, that there is value to having a broader array of interests represented in a legislature. If that is the case, then an ideal system is one in which the shape of the distribution of ideology scores in a state delegation is identical to the shape of the distribution of ideology scores in the electorate. Also, the median voter of the state delegation should have the same ideology as the median voter in the state, and the variance in ideology in the legislature should be as close as possible to the variance in ideology among the electorate. In this section, I will estimate the variance in the ideology scores of the state delegation under the competitive plan, and under the non-competitive plan. I will also discuss the shape of distributions under the competitive and the non-competitive plan.

Let us first analyze the performance of competitive districts. Recall that for any district i, the general election winner will have an ideology of d_i, which is a random draw from the following distribution:

$$d\begin{cases} d_i = d' + \varepsilon & \text{with prob. of .5} \\ d_i = -d' + \varepsilon & \text{with prob. of .5} \end{cases} \quad \text{where } d' > 0, |d_i| \leq D \text{ and } E(\varepsilon) = 0$$

Thus, it is not possible for the competitive plan to produce a state delegation with an ideological distribution that is standard normal because scores are bounded. Even if d had a different distribution, it must be bounded by D and $-D$ because of the requirements of Nash equilibria. In fact, an interesting irony here is that the smaller D is, the better competitive districts do with respect to Goals 1 and 3 (after all, if D is arbitrarily close to 0, legislators are constrained to be arbitrarily close to their district medians, and the median legislator is constrained to be arbitrarily close to 0), but the worse they do with Goal 4 because it makes the difference between the distribution of ideology scores in the state delegation and the standard normal distribution more dramatic. Along similar lines, recall that it is possible (although unlikely) for the competitive plan to meet Goal 1, or, for all legislators to be at their district medians. Even if this unlikely outcome occurred, note that the variance in ideology scores for the state delegation would be 0. In that case, the competitive plan would perform very poorly with respect to Goal 4.

So, the competitive plan cannot produce a state delegation with a standard normal distribution. However, can it produce something similar? The variance in ideology scores for the state delegation will be $(1/(n-1))\sum_{i=1}^{n}(d_i)^2$. We can approximate that with $(1/(n-1))\sum_{i=1}^{n}(d_i)^2 = (n(d')^2/(n-1))$, which will be bounded above by some arbitrary value of D^2, below by 0, and converge to some arbitrary value, $(d')^2$ as the number of districts increases. This could be greater than 1, or it could be less than 1. Without further assumptions, we cannot say. However, suppose $d' = ((n-1)/n)^{1/2}$. If that is the case, then $(1/(n-1))\sum_{i=1}^{n}(d_i)^2 = (n(d')^2/(n-1)) = 1$. This is not implausible since $((n-1)/n)^{1/2}$ approaches 1 as n approaches infinity, and $d' = 1$ is reasonable since 68 percent of the state is between -1 and 1. However, there is no a priori reason to believe that this is the case, and even if it is, there are still two problems: the distribution is bimodal rather than unimodal (and regardless of the distribution of d, it is at least bounded), and legislators are deviating from their district medians by a large amount! The latter problem is true regardless of the distribution of d since a variance of 1 by necessity means that legislators deviate from their district medians under the competitive plan.[24] Thus, the competitive plan cannot fully meet Goal 4. Even if the competitive plan manages to partially accomplish Goal 4 by achieving a state delegation with a variance of 1, it is at the expense of Goal 1. On the other hand, if the competitive plan manages to accomplish Goal 1 (which is unlikely), then the variance in ideology scores would be 0, and the plan would perform quite poorly with respect to Goal 4. The competitive plan cannot meet Goal 4 very well, and it cannot simultaneously meet Goal 1 and even partially meet Goal 4.

Now, let us consider the non-competitive districts. Consider two arbitrary truncation lines, x_a and x_b: $F^{-1}(a/n)$ and $F^{-1}(b/n)$ where $a > b$. The proportion of the population in this range is $\int_{F^{-1}(b/n)}^{F^{-1}(a/n)} f(x)dx$. This function is easily simplified:

$$\int_{F^{-1}(b/n)}^{F^{-1}(a/n)} f(x)dx = F\left(F^{-1}\left(\frac{a}{n}\right)\right) - F\left(F^{-1}\left(\frac{b}{n}\right)\right)$$

$$F(F^{-1}(a/n)) - F(F^{-1}(b/n)) = (a/n) - (b/n) = (a-b)/n$$

Thus, $\int_{F^{-1}(b/n)}^{F^{-1}(a/n)} f(x)dx = ((a-b)/n)$. However, $(a-b)/n$ is also the proportion of legislators with ideology scores in-between $F^{-1}(a/n)$ and $F^{-1}(b/n)$. This means that the proportion of legislators with ideology scores between any two truncation lines is the same as the proportion of the population with ideology scores between any two truncation lines. This

24. Recall that the average ideology score of each competitive district is 0.

is precisely what proportional representation means. Furthermore, as discussed earlier, with infinitely many districts, each district represents a single ideology score, and each legislator has the ideology score of his district. So, let $g(x)$ be the probability density function of legislators' ideology scores under the non-competitive plan, and $f(x)$ be the p.d.f. of the standard normal distribution. By the same logic as above, $\int_b^a f(x)dx = \int_b^a g(x)dx$. Therefore, $f(x) = g(x)$. Thus, with infinitely many districts, the distribution of legislators' ideology scores is the same as the distribution of the state's ideology scores, and the non-competitive plan meets Goal 4. The smaller the districts, i.e. the more districts, the more precisely the distribution of scores in the state delegation approximates the distribution of the population.

So, how quickly does the distribution of legislators' ideology scores begin to approximate a standard normal distribution under the non-competitive plan? One way to answer that question is to look at the variance in ideology scores among the state delegation and see how quickly it converges to 1, which is the variance of the population's ideology scores. First, we can compute the variance in ideology scores if every legislator is at his district median. With finitely many districts, no legislators are actually at their district medians, as discussed earlier, but ideology scores quickly converge to district medians, as shown in the discussion of Goal 1. The general formula for the median of district i is as follows: $\text{median}_i = F^{-1}((i/n) - (1/2n))$. Consequently, the estimator for the variance of the medians is:

$$\sigma^2_{leg\,\cdot\,median} = \frac{1}{n-1}\sum_{i=1}^{n}\left(F^{-1}\left(\frac{i}{n} - \frac{1}{2n}\right)\right)^2.$$

Now, let us consider the lower bound of the variance under the non-competitive plan. Under a non-competitive redistricting plan, the value of σ^2_{leg} is bounded below by what the variance would be if each district elected a representative at the upper limit of its distribution for left-of-center districts, and a representative at the lower limit of its distribution for right-of-center districts, i.e. if every district elected someone at its end closest to the state mean. If that is the case, then the variance of ideology scores in the state delegation would be the sum of the squares of the boundaries divided by the number of districts minus 1, or,

$$\sigma^2_{leg\,\cdot\,\min} = \frac{1}{n-1}\sum_{i=1}^{n-1}\left(F^{-1}\left(\frac{i}{n}\right)\right)^2.^{25}$$

25. It is an odd mathematical property that even though there will be n districts, the series only needs $n-1$ elements. If the number of districts is even, there are two districts that will elect legislators at ideology 0, which don't need to be double counted, so the series is from 1 to $n-1$. If the number of districts is odd, the center district elects a legislator at ideology 0, which doesn't need to be added into the series since $(0-0)^2 = 0$, so the series is still from 1 to $n-1$.

Finally, while there is no true upper bound for variances, we can estimate a practical upper bound for variances under non-competitive districts if we assume that each non-tail district elects a legislator at its most extreme end, and each tail district elects a legislator whose ideology score is more extreme than precisely 3/4 of its voters.[26] In that case, the variance estimator would be:

$$\sigma^2_{leg.\max} = \begin{cases} \frac{1}{n-1}\left(2\left(F^{-1}\left(\frac{1}{4n}\right)\right)^2 + \sum_{i=1}^{n-1}\left(F^{-1}\left(\frac{i}{n}\right)\right)^2\right) \text{ if } n \text{ is even} \\ \frac{1}{n-1}\left(2\left(F^{-1}\left(\frac{1}{4n}\right)\right)^2 + \left(\sum_{i=1}^{n-1}\left(F^{-1}\left(\frac{i}{n}\right)\right)^2\right) - \left(F^{-1}\left(\frac{1}{2}+\frac{1}{2n}\right)\right)^2\right) \\ \text{if } n \text{ is odd.}[27] \end{cases}$$

Now, we can plot these variance estimators for any given number of districts up to 53, as before. These variance estimators are plotted in Figure 3.

It appears that the variance in ideology scores of the state delegation under the non-competitive plan very quickly converges to 1. While the upper and lower bounds provide some leeway, recall that legislators' ideology scores quickly converge to their district medians under the non-competitive plan, and the variance in district medians very quickly approaches 1.

We can now draw a very clear comparison between the variance in ideology scores for the state delegation under competitive vs. non-competitive districts. Under the non-competitive plan, as the number of districts increases, the distribution of ideology scores among the state delegation converges to a standard normal distribution very quickly. Thus, the non-competitive plan fulfills Goal 4 by achieving 'proportional representation' very well. On the other hand, the competitive plan cannot produce a state delegation with a standard normal distribution of ideology scores. Furthermore, it cannot produce a state delegation with a variance of 1 without sacrificing Goal 1, and if it achieves Goal 1, its state delegation will have a variance of 0. Thus, even if it partially fulfills Goal 4, it is at the expense of Goal 1 (and fulfilling Goal 1 is at the expense of Goal 4). Furthermore, the shape of the distribution under

26. This cut-off is, of course, arbitrary, but assumes a very large magnitude difference between the ideology of the median voter and the legislator since voters in the tail districts are very spread out anyway. As such, this model probably over-estimates the practical upper bound, but the results will still converge to 1 anyway.

27. If n is even, the series counts $n+1$ squares instead of n. However, since one of those counts is a 0 that doesn't actually have any legislators at that point, it doesn't affect the calculation. When n is odd, one of the squares must be subtracted because summing up the squares of the boundaries double-counts the center district.

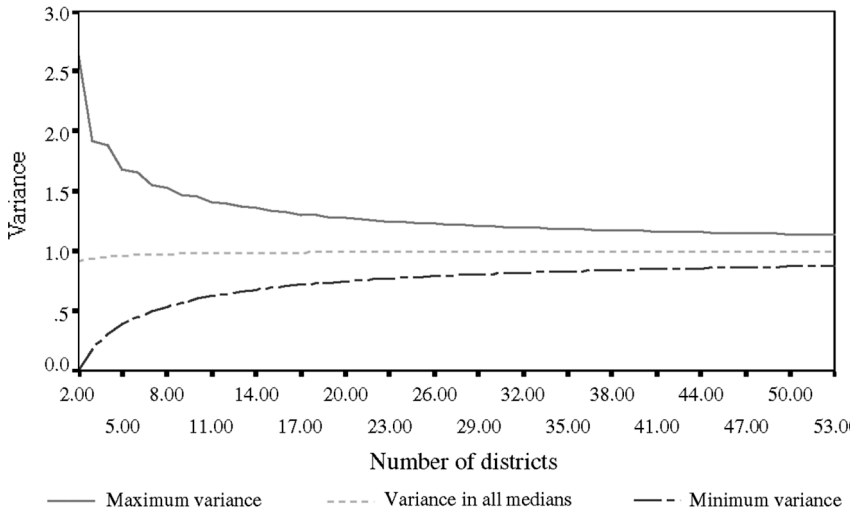

Figure 3. Variance in Ideology Scores for State Delegation under Non-competitive Redistricting Plan

the competitive plan will likely be bimodal rather than unimodal, and there is no a priori reason to believe that it will have a variance of 1. Thus, it could have a variance that is too low, indicating that voters far from the median are under-represented, or it could mean that the variance is too high, meaning that centrists are under-represented. In fact, a bimodal distribution with a variance less than 1 would have a unique irony – it under-represents both centrists and those far from the median! On the other hand, a bimodal distribution with a variance greater than 1 would have a different but equally puzzling irony – it would be too heterogeneous, but still under-represent a key group: centrists! Thus, the non-competitive plan clearly out-performs the competitive plan with respect to Goal 4, and it does so while minimizing the degree to which legislators can deviate from the positions of their district medians (i.e. while meeting Goal 1) – a balance which the competitive plan cannot meet.

Once again, though, it is important to note that these results are not generated by the truncated shape of the districts. They are the natural consequence of intentionally creating districts whose median voters are shifted away from the median voters of the state. Essentially, voters who are relatively far from the median are ideological minorities, and just as creation of majority–minority districts ensures the representation of under-represented minorities by creating homogeneous districts, the creation of ideologically homogeneous districts ensures the representation of those with different

ideologies. Similarly, if competitive districts lead to under-representation of centrists, drawing overly centrist districts permits them representation. An additional analogy might be made here to the fact that third parties cannot win elections in a single member district system unless they are geographically concentrated. Unless voters with the same ideologies are concentrated, they will lose out to either state-centrists or primary-centrists.

Discussion of Results

This paper began with the following question: if competition is important for democracy because it aggregates preferences, then how representative are those aggregations? The core result of this model is that ideologically homogeneous districts structured in such a way as to minimize competition in congressional elections can do a better job than competitive districts in achieving representative outcomes. These non-competitive districts produce legislators that are closer to their district medians and more representative of everyone in their districts (including those far from the district median). This non-competitive plan also produces a state delegation whose median legislator is more likely to be at the position of the state's median voter, and a distribution of ideology scores among the state delegation that is more similar to the distribution of ideology scores among the population. Furthermore, the non-competitive plan can meet all goals simultaneously, but the competitive plan cannot. It is possible, although improbable, for the competitive plan to prevent legislators from deviating from their district medians, but under those circumstances the competitive plan would fail to have any variation in ideology scores among the state delegation. It is also possible for the competitive plan to produce a state delegation with the same variance in ideology scores as the population, but only when legislators deviate from the positions of their median voters, thus failing to meet the first goal, and even then, the shape of the distribution would be wrong. Thus, it is not possible for the competitive plan to prevent legislators from deviating from their district medians and have the optimal amount of variation in ideology scores among legislators. Both criteria can be met by the non-competitive plan described in this paper. In short, the paradoxical finding of this paper is that competitive elections do not inherently lead to more representative outcomes than non-competitive elections. In fact, competitive districts may produce more unrepresentative outcomes, not because competition leads to unrepresentative outcomes by itself, but because the characteristics that produce competition – political/ideological heterogeneity – also produce unrepresentative outcomes.

These results are certainly counterintuitive, and should cause us to question what it is we want our electoral system to achieve. If we want a system

that promotes a lively debate over large ideas, congressional elections structured non-competitively will certainly not help to promote those debates since congressional candidates facing each other would not disagree with each other on large issues (the only competition would be in primaries in which the candidates are in agreement). Such debates would be relegated to executive elections. However, if the goal is merely to create a legislature that is as representative as possible, competitive elections are not the best method according to these models. The intention of this paper is not to argue that the bipartisan gerrymander is necessarily a good approach – the intention is to argue that the competitive plan is not a particularly good approach, or, at the very least, that we can do better than the competitive plan.

Potential Criticisms

There are a number of reasons to question the conclusions of these simple models. Like all spatial voting models, they are subject to the criticism that voters cannot be characterized as rational utility-maximizers with respect to ideology, particularly since voters empirically have little understanding of what ideology is, as Converse (1964) might argue. A less extreme argument could be made that the results depend on unidimensional ideology scores. However, rather than altogether discarding a vast array of spatial voting models that have proven analytically useful, it would be more fruitful to discuss the particulars of the model. In that vein, a more salient criticism of this model is that it assumes a great deal of precision in a redistricter's ability to group voters into ideologically homogeneous districts. When drawing district lines, the data available consist of demographic figures provided by the census which cover race, income, age, etc., and aggregate voting patterns by precinct. These data would certainly allow a redistricter to create districts that are more homogeneous than the state as a whole, though, and the occurrence of bipartisan gerrymanders such as the 2000 California redistricting are empirical proof of that. Again, recall that the results of these models are not generated by the truncation of the distributions. They are generated by the shifting of district medians away from the state median, and creating districts whose ideological variances are smaller than the ideological variance of the state. Those goals are certainly achievable by a redistricter. As districts become more homogeneous, representative goals become easier to achieve, and redistricters certainly have the tools to make districts more homogeneous.

A related criticism is that since such homogeneous districts are impossible to draw, there will be voters who are placed in the 'wrong' district. For example, a group of conservative Republicans may find themselves in a district dominated by liberal Democrats. As a result, they are represented by a

legislator who is even farther from them than a centrist, and caught in a district in which they have no chance of winning. While this cannot realistically be called 'disenfranchisement', since doing so would mean that anyone whose candidate doesn't win is disenfranchised, it does mean that some voters would be worse off than they would be under competitive redistricting plans with respect to Goal 2 of this paper (descriptive representation at the district level).

Other criticisms may be made on the basis of the fact that the non-competitive redistricting plan may come into direct conflict with other redistricting objectives (aside from the obvious point that many consider the creation of competitive districts to be a legitimate goal of an ideal redistricting plan). Most obviously, in order to draw lines so as to make districts as homogeneous as this paper suggests, lines would frequently be so bizarre as to make Elbridge Gerry's salamander look like a model of compactness. Alternatively, while ideologically homogeneous redistricting may seem perfectly consistent with the goal of keeping communities of interest together, if communities of interest are ideologically or demographically heterogeneous, the goals again come into conflict.

Another criticism might be that the model in this paper misrepresents the problem with non-competitive elections. Perhaps the problem is not the potential for legislators to be unrepresentative, but that they might shirk in terms of participation. Rothenberg and Sanders (2000) demonstrate that when legislators are going to retire, they do not change their ideological voting patterns, but may be less likely to actually cast votes. Similarly, it is possible that systematically eliminating competitive congressional elections may reduce legislative activity by removing the incentives. That is an open question, and while Groseclose's (2001) findings and others on valence issues might suggest otherwise, it is a legitimate concern. However, it probably doesn't rise to the level of concern over ideological representativeness because unless there is a bias in participation, outcomes should be the same.

Perhaps the greatest criticism of this model, though, might come from its reliance on effective congressional primaries. What this paper refers to as a non-competitive redistricting plan is not one that abolishes all competition, but one that forces all meaningful competition out of the general elections and into the primaries. Thus, if elections are control mechanisms that allow voters to have power over their representatives, that mechanism must be moved to the primary stage. However, congressional primaries attract very little attention from voters, and incumbents rarely face competitive primaries. Given voters' lack of attention to primaries, it may be necessary to reconsider the assumption that no district can either nominate or elect a legislator outside the ideological range of the district. It would not be difficult to imagine, for example, a centrist but partisan district electing a representative who is fairly extreme because the extremist candidate is

the only qualified candidate. Since congressional primaries attract very little attention, such a scenario is not outside the realm of possibility.

A system in which the primary election is the important one is not new to this country, though. In the South, between Reconstruction and the recent rise of a Republican Party in the South, the Democratic primaries were the only campaigns that mattered, and voters knew it. Hence, their attention was focused on primaries. However, it would be difficult to imagine a parallel situation occurring today. In order for that to happen, voters would have to focus on Congress during the primaries, and the Presidency during general elections. Given that voters do not pay a great deal of attention to congressional elections during mid-term elections, it is not necessarily reasonable to expect them to divide their attention so effectively.

Given these concerns, it may not be ideal to actually attempt to structure congressional elections in a manner prescribed by this paper, and again, this model is not intended to provide a policy prescription. However, if we can call into question the ability of competitive elections to achieve the goals of representative democracy, we must consider more closely what we want our electoral system to achieve. What do we really want our electoral system to do? I do not have an answer. It is my hope, though, that results such as these will motivate people to address the question more carefully.

Other Arguments against Competition

The results of this model should lead us to question whether or not competitive elections are truly important for a democracy. However, the idea that having competitive elections produces problems is not new. With respect to redistricting, as discussed earlier, Niemi and Deegan (1978) and Niemi (1982) show that in choosing a redistricting plan, the goal of promoting competition can come into conflict with the goals of neutrality (lack of partisan bias), ensuring a constant swing ratio (keeping vote changes proportional to seat changes), and ensuring that the responsiveness range does not allow one party to be unrepresented. Along similar lines, Butler and Cain (1992) show empirically that a system of competitive elections does not lead to optimal partisan representation. The results of this model complement those findings very well.

Even beyond the concept of representation, arguments can certainly be made that districts should not be uniformly competitive. For example, if one were to accept the goal of creating majority–minority districts to elect minority representatives, that would mean advocating districts that are overwhelmingly Democratic, and thus not competitive. Similarly, keeping communities of interest together is anti-competitive when those communities are strongly partisan. Many more of such tradeoffs are discussed in the works previously mentioned.

Outside of redistricting, we can find still more arguments that competition is not always beneficial. Most obviously, the presence of competitive elections for MCs every two years can be a distraction from the business of legislation. Legislators, who fear losing their positions even when they are relatively safe (Mann, 1978), spend a great deal of time raising money and securing reelection at the expense of legislating. On issues of low public salience, this fear of losing increases a legislator's willingness to 'sell votes' because the need for money is greater than the potential political cost of selling the vote (Buchler and Jarvis, 2001). Even in the absence of corruption, the reelection motive can lead legislators to focus on parochial interests, such as constituency service and particularized benefits to narrow groups rather than on broad issues of national importance (Mayhew, 1974; Fiorina, 1989). Thus, factors serving to decrease competition are a product of the threat of competition.

Additionally, we might ask why any given district should have close elections. Let us take a delegate model of representation in which a representative's job is to replicate the preferences of his constituents. If such patterns occur because voters elect representatives who share their opinions, competition is not necessary to keep the representative in line because he will simply follow his own preferences, which happen to be the preferences of his constituents. If elections in his district are competitive, the district risks losing an ideal representative in exchange for one who cannot be much better, and could easily be worse. Even if we assume that representatives take their positions due to constituency pressure, if the threat of competition successfully keeps legislators in line, then they should never face competitive elections, just as if police deter all crime, no one would ever be arrested. After all, incumbents may win by large margins, but they do so because, as Fenno's paradox goes, constituents tend to like their representatives, even if they dislike Congress in general (Fenno, 1978). Thus, the lack of competition may indicate that the system is working rather than failing. After all, Congress is structured so that districts can gain distributive benefits from having senior representatives, and it may be beneficial in general to have legislators who are experienced.

Given those arguments in addition to the findings of this paper, it is well past time for political scientists and reformers to have a serious discussion about what goals we should attempt to achieve in electoral reform.

Empirical Questions and Directions for Future Research

While the primary goal of this paper is to spark a normative discussion of what we want from an electoral system, the model in this paper also points toward some possible empirical research. This model generates several

potential hypotheses to test. The two most obvious questions that beg for empirical testing are as follows: First, is it true that legislators deviate less frequently from the positions of their district medians when their districts are homogeneous? Similar hypotheses have been tested with respect to Senate constituencies (e.g. Bailey and Brady, 1998), but since districts can be drawn more homogeneously than states, the question remains open with respect to the House. Second, is it true that states with less competitive districts have delegations with ideological distributions that look more like their populations? If the predictions made by the model in this paper are true, the implications are most obvious for questions of House–Senate relations. Since the Senate cannot be gerrymandered to maximize representativeness, such a difference across legislative bodies has potentially far-reaching implications for House–Senate relations.

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